Freshmen Programming Contests 2024

Solutions presentation

By the Freshmen Programming Contests 2024 jury for:

- AAPJE in Amsterdam
- FPC in Delft
- FYPC in Eindhoven
- GAPC in Groningen

May 4, 2024

A: Annoying Alliterations

Problem Author: Maciek Sidor

- **Problem:** Given n words, find a pair such that after their common prefix is removed, the sum of lengths of the two resulting words is the greatest.
- **Naive solution:** Check every pair. Runs in $\mathcal{O}(n^2)$, too slow.
- **Claim:** For a given pair s, t and a third word v such that $|v| \geq \max(|s|, |t|)$, we can always replace one of the words and the score will not decrease.

- **Claim:** For a given pair s, t and a third word v such that $|v| \geq \max(|s|, |t|)$, we can always replace one of the words and the score will not decrease.
- **Proof:** Denote the common prefix of s, t as $p(s, t)$ and let $g(s, t) = |s| + |t| - 2|p(s, t)|$ be our goodness function.
- Suppose $g(s, t) > g(s, v)$ and $g(s, t) > g(v, t)$. Then:

$$\begin{aligned} |s| + |t| - 2|p(s, t)| &> |s| + |v| - 2|p(s, v)| \\ |t| - 2|p(s, t)| &> |v| - 2|p(s, v)| \\ |p(s, v)| &> |p(s, t)| \end{aligned}$$

- Similarly, $|p(v, t)| > |p(s, t)|$, but these two together give us a contradiction. $\square$

A: Annoying Alliterations

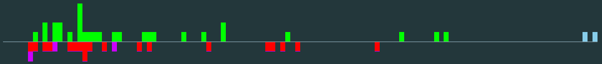
Problem Author: Maciek Sidor

- **Claim:** For a given pair s, t and a third word v such that $|v| \geq \max(|s|, |t|)$, we can always replace one of the words and the score will not decrease.
- **Therefore:** Any word of maximum length is part of a valid solution.
- **Solution:** Pick any word of maximum length and check it with every other word, take the maximum result.
- **Complexity:** $\mathcal{O}(n)$.
- **Note:** Can also be solved using a trie (also known as a prefix tree).

Statistics: 23 submissions, 1 accepted, 6 unknown

B: Building Pyramids

Problem Author: Maarten Sijm



- **Problem:** Given the edge length of a tetrahedron, calculate the number of spheres in the pyramid.
- **Observation:** The pyramid consists of n equilateral triangles.
  The number of spheres in triangle t is $T(t) = \sum_{i=1}^{t} i$.
- **Slow solution:** Calculate $P(n) = \sum_{i=1}^{n} T(i)$. Runs in $\mathcal{O}(n^2)$, too slow.
- **Solution:** Simplify $T(t) = \frac{t \cdot (t+1)}{2}$. Now calculating $P(n)$ runs in $\mathcal{O}(n)$, accepted!
- **Pitfall:** If t is an `int`, $t \cdot (t+1)$ overflows. Use 64-bit integers!
- **Challenge:** The calculation of $P(n)$ can even be simplified to run in $\mathcal{O}(1)$.

Statistics: 53 submissions, 29 accepted, 2 unknown

C: Curious Jury

Problem Author: Jeroen Op de Beek

- **Problem:** Given two types of penalty times for n teams ($1 \leq l_i < s_i \leq n$), find out over all ways of choosing the type of penalty time for each team, how many fixed points the scoreboard contains in total.
- **Observation 1:** Instead of finding fixed points for each out of 2^n options, find how many times team i is a fixed point.
- **Observation 2:** Loop over all teams, try both options l_i and s_i as potential fixed point, for team i, call this potential fixed point f.
- **Observation 3:** Other teams form 3 groups:
  - **A** Teams with $l_j < f, s_j < f$
  - **B** Teams with $l_j < f, s_j \geq f$
  - **C** Teams with $l_j \geq f, s_j \geq f$
- **Observation 4:** The number of ways to choose the other submission times, for team i to have a fixed point at rank f: $2^{|A|+|C|} \cdot \binom{|B|}{f-|A|}$
- Team j is in group **A** if $s_j < f$.
- Team j is in group **C** if $l_j \geq f$.
- Otherwise, team j is in **B**. By sorting the l_j and s_j arrays, $|A|$ and $|C|$ can be found by binary search.

C: Curious Jury

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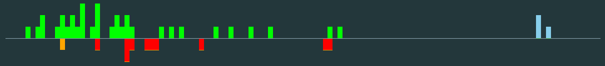
- **Problem:** Given two types of penalty times for n teams ($1 \leq l_i < s_i \leq n$), find out over all ways of choosing the type of penalty time for each team, how many fixed points the scoreboard contains in total.
- **Observation 4:** The number of ways to choose the other submission times, for team i to have a fixed point at rank f: $2^{|A|+|C|} \cdot \binom{|B|}{f-|A|}$
- Need to calculate $\mathcal{O}(n)$ binomial coefficients $\binom{a}{b}$, with $0 \leq a, b \leq n$: $\binom{a}{b} = \frac{a!}{b!(a-b)!}$ and 2^a for $0 \leq a \leq n$, both $\mod (10^9 + 7)$
- Can precalculate factorial[k] and twopower[k] in $\mathcal{O}(n)$.
- Can find inverse of factorial[n] in $\mathcal{O}(\log(MOD))$ (or if you don't know how to calculate a modular inverse, you can bruteforce it on your own computer).
- Now fill the array invfactorial[k] using invfactorial[k] $=$ invfactorial[$k+1$] $\cdot (k+1)$ in $\mathcal{O}(n)$.
- Complexity varying from $\mathcal{O}(n(\log(n) + \log(MOD)))$ to $\mathcal{O}(n + \log(MOD))$ depending on exact implementation.

Statistics: 3 submissions, 0 accepted, 3 unknown

D: Dragged-out Duel

Problem Author: Wietze Koops

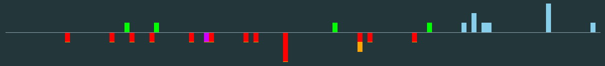


- **Problem:** Read two lines, comprised of '`R`', '`P`', and '`S`', and determine who wins the most games.
- **Solution:**
  - Read the two lines character by character, increment a counter if player 1 wins and decrement it if player 2 wins.
  - Finally, print "`victory`" if the counter is positive, and "`defeat`" if it is negative.
- **Complexity:** $\mathcal{O}(n)$.

Statistics: 50 submissions, 34 accepted, 3 unknown

E: European Election

Problem Author: Veselin Mitev



- **Problem:** Given ranked-choice ballots, determine the candidate who beats all other candidates.
- **Fun Fact:** This is also known as the *Condorcet* voting method.
- How to determine whether candidate A is better than B?
  - Go through all ballots – count how many times A appears before B, and vice-versa. Runs in $\mathcal{O}(n \cdot k)$.
- **Observation:** We can preprocess the ballots in $\mathcal{O}(n \cdot k)$, such that we can access the position that each candidate appears in each ballot in $\mathcal{O}(1)$. Thus, answering whether candidate A beats candidate B, now only takes $\mathcal{O}(n)$.
- **Solution:**
  - Pick a candidate d.
  - Go through all candidates c_i: Anytime c_i beats d: $d \leftarrow c_i$. Runs in $\mathcal{O}(n \cdot k)$.
  - **Observation:** If the election has a winner, it must be d. (This can be proven using contradiction!)
  - Check if d beats all other candidates. Runs in $\mathcal{O}(n \cdot k)$.
- **Complexity:** $\mathcal{O}(n \cdot k)$.
  - But we chose the time limit to also accept $\mathcal{O}(n \cdot k^2)$ or even $\mathcal{O}(n \cdot k^3)$.

Statistics: 29 submissions, 4 accepted, 9 unknown

F: Flag Rotation

Problem Author: Jeroen Op de Beek

- **Problem:** Count how many cells will change when painting the flag rotated.
- **Observation:** Since each column has to be repainted to one color, we will change $n - cnt_c$ cells in it (where c is the final color).
- **Solution:** Count how many cells won't change.
  - First sort the array, then check for segments made of identical elements, this way we find the count of each cell color.
  - answer $= n^2 - \sum_c cnt_c^2$
- **Complexity:** $\mathcal{O}(n \log n)$.
- **Note:** this can also be done using a (hash) map.

Statistics: 76 submissions, 20 accepted, 16 unknown

G: Galactic Expedition

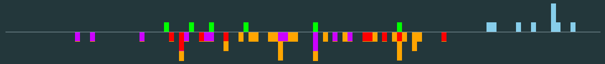
Problem Author: Veselin Mitev

- **Problem:** Navigate between wormholes to find the ancient relic, without running out of fuel.
- **Observation:** You can refuel more than enough times to simply explore all wormholes, until you find a way to reach the relic.
- **Solution:** Perform a “live” search – explore the wormholes while always keeping enough fuel ($\frac{d}{2}$) to go back to home base:
  - If you can reach the relic within the fuel limit, do that.
  - Find the closest unexplored wormhole:
    - You can do that using **Dijkstra**, or **Floyd-Warshall**.
  - Can we reach it while still having enough fuel to go back to home base?
    - If yes: Go to that wormhole and update the distances between the points.
    - If no: Go back to home base and refuel.
  - Repeat.
- Worst case: We can explore all wormholes in $\frac{n}{2}$ runs.
- If we know all wormholes, it is guaranteed that we can reach the relic, if we follow an optimal path.
- **Time Complexity:** $\mathcal{O}(n^3)$ or $\mathcal{O}(n^3 \log n)$. Or if you're clever about how you cache the results from the Dijkstra search algorithm you can do it in $\mathcal{O}(n^2)$ or $\mathcal{O}(n^2 \log n)$.

Statistics: 0 submissions, 0 accepted

H: Horrendous Mistake

Problem Author: Jeroen Op de Beek

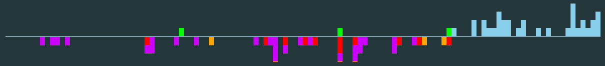


- **Problem:** Calculate the value of the function `sum`, which uses values instead of indices.
- **Naive solution:** Simply run the function after every update. This takes $\mathcal{O}(n \cdot q)$ time, too slow!
- **Observation:** To be fast enough, every query must be processed in $\mathcal{O}(1)$.
- **Solution:** Do some extra bookkeeping:
  - Count how often every value occurs in the initial array ($= c_x$ for every $0 \leq x < n$).
  - Calculate the value of `sum` for the initial array and store this.
  - For every update (x, v) (let the old value in the array be v_{old}):
    - Decrement $c_{v_{old}}$.
    - Subtract $c_x \cdot v_{old} + a_{v_{old}}$ and add $c_x \cdot v + a_v$ to the stored value of `sum`.
    - Increment c_v.
    - Update the value in the array.
- **Complexity:** $\mathcal{O}(n + q)$.
- **Pitfall:** Beware of `int` overflow, be sure to use 64-bit integers!

Statistics: 59 submissions, 6 accepted, 9 unknown

I: Intelligence Exploration

Problem Author: Makar Kuleshov



- **Problem:** Calculate the value of the implication $a_l \to a_{l+1} \to \ldots \to a_r$ for many subarrays.
- **Naive solution:** Go through the whole subarray to calculate the result. Runs in $\mathcal{O}(n \cdot q)$, too slow!
- **Observation:** When the right argument of an implication is 1, the result is always equal to 1.
  So, we can look only at the last 1 in the subarray and the following zeros.

  $a_l \to \ldots \to 1 = 1 \quad \underbrace{a_l \to \ldots \to 1}_{1} \to 0 = 0 \quad \underbrace{a_l \to \ldots \to 1 \to 0}_{0} \to 0 = 1$

  $\underbrace{a_l \to \ldots \to 1 \to 0 \to 0}_{1} \to 0 = 0 \quad a_l \to \ldots \to 1 \to \underbrace{0 \to \ldots \to 0}_{k \text{ zeros}}$

  If k is even then the result equals 1.
  If k is odd then the result equals 0.
- **Solution:** For each position precompute the index of the last 1 appearing not after it.
  This way you can determine the number of zeros in the end of a subarray in $\mathcal{O}(1)$.
- **Complexity:** $\mathcal{O}(n + q)$

Statistics: 76 submissions, 3 accepted, 33 unknown

J: Jailbreak

Problem Author: Wietze Koops

- **Problem:** Escape from a $w \times h$ grid jail where you can go up only if you have a ladder. Ladders can be carried to a different place on the same storey.
- **Observation:** If we know to which holes a ladder can be carried, then for each cell, we know which cell we can move to.
- **Solution:**
  - Using a for loop in both directions, determine which cells can access a ladder.
  - Then we know for each cell to which cell we can move.
  - Hence, we can define a graph representing the grid.
  - Determining whether a path exists from the starting cell to an exit can be done using $\mathcal{O}(wh)$ BFS/DFS.

Statistics: 18 submissions, 0 accepted, 16 unknown

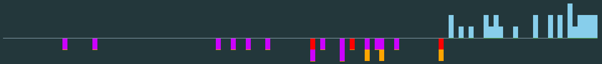
K: Kangaroo Race

Problem Author: Leon van der Waal

- **Problem:** A kangeroo jumps from x to $x + x(x-1) = x^2$ in one step. How many steps until it reaches 1?
- Notice that when a kangeroo jumps over the n-th segment, it jumps to $x^2 \mod n$.
- So after i jumps, the kangeroo is in segment $x^{2^i} \mod n$.
- Therefore we need to determine the first i such that $x^{2^i} \equiv 1 \mod n$.

K: Kangaroo Race

Problem Author: Leon van der Waal



- **Problem:** Determine the first i such that $x^{2^i} \equiv 1 \mod n$.
- **Observation:** If $\gcd(x, n) \neq 1$, then $\gcd(x, n) \mid x^{2^i} \mod n$, so the kangeroo will never reach 1.
- Otherwise, $x^r \equiv 1 \mod n$ for some r. We call the smallest such r the *order* of x mod n.
- Notice that the powers of x repeat every r-th power:

$$1, x, x^2, x^3, \ldots, x^{r-1}, x^r = 1, x^{r+1} = x, x^2, x^3, \ldots$$

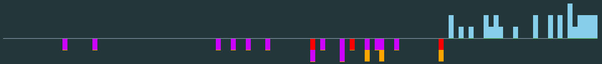
K: Kangaroo Race

Problem Author: Leon van der Waal

- **Problem:** What is the first i such that $x^{2^i} \equiv 1 \mod n$.
- Therefore, any i that satisfies $x^{2^i} \equiv 1 \mod n$ also satisfies $2^i \equiv 0 \mod r$.
- **Observation:** This means that r is a divisor of 2^i, and thus $r = 2^k$ for some k.
- Therefore, the answer to the problem is k.
- **Observation:** $r \leq n$, so $k \leq \log_2(n)$

K: Kangaroo Race

Problem Author: Leon van der Waal



- **Problem:** What is the first i such that $x^{2^i} \equiv 1 \mod n$.
- **Observation:** $r \leq n$, so $k \leq \log_2(n)$
- **Solution:** It therefore suffices to check the first $\log_2(n) < 60$ jumps. If the kangeroo has not reached segment 1 by then, it never will.
- **Complexity:** $\mathcal{O}(q \log n)$

Statistics: 49 submissions, 0 accepted, 29 unknown

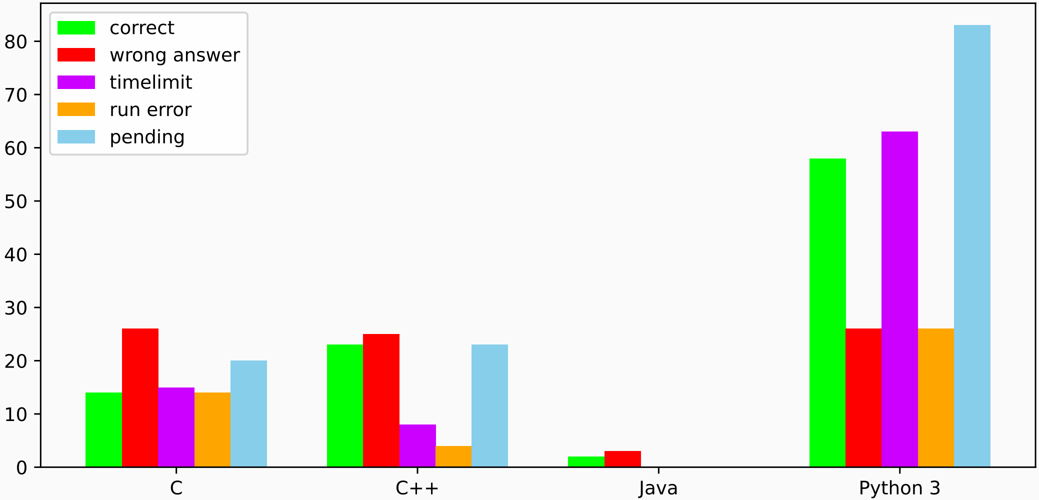
Open online contest

Want to solve the problems you could not finish?
Or have friends that like to solve algorithmic problems?

`https://fpcs2024.bapc.eu/`

Friday 10 May 2024 13:00–17:00

Language stats



Random facts

Jury work

- 447 commits (last year: 361)
- 357 secret test cases (last year: 339)
- 120 accepted jury/proofreader solutions (last year: 96)
- The minimum[1] number of lines the jury needed to solve all problems is

$$2 + 1 + 11 + 1 + 5 + 1 + 22 + 5 + 3 + 11 + 4 = 66$$

On average 6.0 lines per problem, down from 6.4 last year

[1] *After* codegolfing

Thanks to the proofreaders:

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- Nicky Gerritsen
  (TU Eindhoven / DOMjudge)
- Pavel Kunyavskiy
  (JetBrains Amsterdam, Kotlin Hero 🎈)
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  (ETH Zürich / NWERC jury)
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